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PAPER NO. 994

## The Present Value Model of Stock Prices: Empirical Tests Based on Instrumental Variables Estimators

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Empirical Tests Based on Instrumental Variables Estimators

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## ABSTRACT

The variance bounds tests of the present value model of stock prices are re-examined in this paper. A direct test of the model based on generalized instrumental variables estimation is proposed as an alternative and it is shown that this approach has several advantages over the variance bounds tests. The test based on instrumental variables estimation is modified to handle the case in which the percentage changes in real dividends and real stock prices are stationary processes. The tests are applied to quarterly data for the Standard & Poor's Index of 500 Common Stocks and the results are much more conclusive than those obtained by the previous tests.



THE PRESENT VALUE MODEL OF STOCK PRICES:  
EMPIRICAL TESTS BASED ON INSTRUMENTAL VARIABLES ESTIMATORS

Shiller (1981a and 1981b) and LeRoy and Porter (1981) have tested the present value model of stock prices by examining the implicit restrictions on the variation of stock prices. Their results suggest that actual stock prices vary too much to be consistent with this model. If we examine their results closely, we find that Shiller does not construct formal statistical tests of the model and that most of the tests in LeRoy and Porter are not statistically significant because the standard errors of the variance estimates are quite large. Some critics have argued that the relevant time series are not covariance stationary, even after the removal of a time trend, and that the variance estimates are therefore unreliable. An alternative is to assume that the percentage changes in dividends, earnings, and stock prices are covariance stationary time series. Shiller (1981a) considers this alternative assumption, but notes that it does not lead to tractable variance bounds for stock prices. In this paper, a test of the present value model based on an instrumental variables estimator is developed as an alternative to the variance bounds test, and the test is extended to handle the case in which the percentage changes in dividends, earnings, and stock prices are covariance stationary. This latter specification requires the use of Hansen's (1982) generalized method of moments estimator.

I. Variance Bounds in the Present Value Model of Stock Prices

The present value model of stock prices has the following form:

$$P_t = E_t \left\{ \sum_{j=1}^{\infty} \beta^j D_{t+j} \right\},$$

where  $P_t$  is the asset price at the end of period  $t$  and  $D_t$  is the dividend paid during period  $t$ .  $E_t$  is the conditional expectations operator, conditional on information available in period  $t$ .  $E$  without a subscript will represent the unconditional expectations operator.  $P_t$  and  $D_t$  are expressed in real terms. This simple expectations model follows from the expected real rate of return being constant and is generally associated with risk neutrality in asset pricing models.<sup>1</sup> If we let

$P_t^* \equiv \sum_{j=1}^{\infty} \beta^j D_{t+j}$ , we observe that the model implies

$$(1) \quad P_t^* = P_t + \eta_t,$$

where  $\eta_t$  is the forecast error. The assumption of rational expectations (or market efficiency) requires that  $P_t$  and  $\eta_t$  be uncorrelated, so that we get

$$\text{Var}(P_t^*) = \text{Var}(P_t) + \text{Var}(\eta_t),$$

and we conclude that  $\text{Var}(P_t) \leq \text{Var}(P_t^*)$ . Thus, we have an upper bound on the variation of stock prices.

Two problems are encountered when one attempts a test of this bound on the variation of stock prices. First, the time series must be covariance stationary; that is, the variances and covariances of the series must be finite and must not depend on time. Shiller removes a long-term trend from his series on dividends and prices and applies the variance restrictions to the detrended series. LeRoy and Porter argue that there are no apparent trends in their adjusted series for earnings and prices and that further adjustments are not necessary. The second problem

deals with the estimation of the variance of  $P_t^*$  (or a detrended  $P_t^*$ ), which is not observable. Shiller calculates this series recursively by assuming a value at the end of the sample period and then computes the sample variance, but he does not develop a statistical test of the model. LeRoy and Porter formulate and estimate a finite-parameter bivariate time series model to compute the variances. This method requires the researcher to formulate a time series model and then test the variance bounds conditional on the formulated model. Singleton (1980) has shown that the variance of a series like  $P_t^*$  can be estimated in the frequency domain if we have a value for the discount factor.<sup>2</sup> The hypothesis tests that a researcher might develop for these variance estimators will depend on the large sample distribution theory for variance estimators. It is well-known that these large sample distributions depend on the effects of fourth cumulants of the innovations of the process generating the observed time series, and for this reason the distributions typically used for hypothesis testing are not robust if we drop the assumption that the innovations are multivariate normal.<sup>3</sup> It is extremely difficult to estimate the standard errors of the variance estimates if we want to relax the assumption of normally distributed innovations (or the assumption that the fourth cumulants are all zero). The large sample distributions for many of the standard econometric estimators (such as ordinary least squares, generalized least squares, or instrumental variables) do not require an assumption that the innovations or error terms are normally distributed. For this reason, the tests based on instrumental variables estimators developed in the next section impose a less restrictive set of assumptions.

LeRoy and Porter developed several tests for the implied variance bounds. Even though their point estimates indicate rejection of the present value model, the confidence intervals are so large that the model is rejected in only a few of the cases examined. In a chapter of my dissertation (1982, Ch. 5), I calculated frequency domain variance estimators and found the point estimates for the variances of the price series to be many times greater than the estimates for the upper bounds, but the standard errors associated with these estimates are so large that the tests of the model are barely significant at the 5% level. It appears that these variances are not being estimated with much precision. More recently, Flavin (1982) has questioned the finite sample properties of the frequency domain variance estimators. One can hardly argue that these tests of the present value model are conclusive.

## II. An Alternative Test Based on Generalized Instrumental Variables Estimation

In an unpublished paper, Geweke (1979) has shown that regression tests of the simple expectations models can be more powerful than the variance bounds tests. This testing procedure has been the one most frequently applied in other tests of simple expectations models (e.g., tests of forward rates as unbiased predictors of spot rates in foreign exchange markets). The regression tests, however, cannot be applied directly to the present value model of stock prices. If the series  $P_t^*$  were observable, then the relationship in equation (1) could be easily tested by a least squares regression of  $P_t^*$  on a constant and  $P_t$ . To derive a regression model, we need to restate equation (1) in

terms of observable dividends and stock prices. First rewrite the equation under the alternative hypothesis using the lag operator ( $L$ :  $Lx_t = x_{t-1}$ ):

$$\frac{\beta L^{-1}}{1-\beta L^{-1}} D_t = a + bP_t + \eta_t.$$

Now we multiply both sides of the equation by the filter  $\frac{1-\beta L^{-1}}{\beta L^{-1}}$  to get

$$(2) \quad D_t = a - bP_t + \frac{b}{\beta} P_{t-1} + u_t,$$

where  $u_t = (\frac{1}{\beta} L - 1)\eta_t$ . We now have an equation that is a function of observable time series and an error term, but the equation does not satisfy all the assumptions of the classical regression model. The forecast errors  $\eta_t$  represent forecast errors for future dividends and are serially correlated. The error term  $u_t$ , which is a function of the forecast errors, will be serially correlated in most cases. The more important problem is that  $P_t$  and  $u_t$  are correlated; therefore, we must use an instrumental variables (IV) estimator for the parameters of the equation. Under the null hypothesis of the present value model,  $a=0$ ,  $b=1$ , and  $E_t(\eta_{t+j})=0$  for  $j \geq 0$ . Under the alternative hypothesis, we relax the restrictions on  $a$  and  $b$  and preserve the restriction on the forecast errors. Because the error term of equation (2) is a function of  $\eta_t$  and  $\eta_{t-1}$ ,

$$E(D_{t-j} u_t) = E(P_{t-j} u_t) = 0 \quad \text{for } j > 0,$$

but  $E(P_t u_t) \neq 0$ . In addition,  $u_t$  should be uncorrelated with any variable (e.g., earnings) dated  $t-1$  or earlier. A natural set of instruments for this problem should include lagged dividends, the lagged

stock price, and lagged values of variables which are important for predicting dividends. Lintner (1956) and Fama and Babiak (1968) have shown that earnings play an important role in the behavior of dividends over time; therefore lagged earnings should be included.

The relevant time series must have finite second moments in order to estimate the parameters in equation (2), and this condition is normally satisfied by requiring that the time series be stationary, or at least covariance stationary.<sup>4</sup> This condition is necessary for estimating the variances in the variance bounds tests and it is equally important for estimating the parameters in equation (2). One approach is to follow Shiller and remove a long-term trend from the data as follows:  $p_t = P_t(1+g)^{-t}$  and  $d_t = D_t(1+g)^{-t}$ . The discount factor for the model using detrended data becomes  $\gamma = \beta(1+g)$ , and the following equation can be derived for the detrended series:

$$(3) \quad d_t = a - b p_t + \frac{b}{\gamma} p_{t-1} + u_t.$$

For equation (3), we still have the condition that  $E_{t-1}(u_t) = 0$  and we must use an IV estimator. To motivate the alternative hypothesis in which  $a \neq 0$  and  $b \neq 1$ , we can imagine a world in which the stock market correctly discounts the constant and the long-term trend of dividends, but systematically overestimates or underestimates the deviations of dividends about the long-term trend. If the market overestimates these deviations from the long-term trend, then the coefficient  $b$  should have a value less than one indicating that stock prices vary too much. The estimation problem for equation (3) does simplify because it can be shown that the error term for this specification is white noise. This



result is shown in an appendix for the case in which  $d_t$  has a linear multiple time series representation. Because the error term is not serially correlated, we can directly apply Amemiya's (1974) nonlinear two-stage least squares estimator.

In the alternative specification, I assume that the dividend process is not mean-reverting. For this second case, I assume that the percentage change in dividends and stock prices as well as the price-dividend ratio  $(\frac{\Delta D}{D}, \frac{\Delta P}{P}, \frac{P}{D})$  are stationary. Shiller (1981a) notes that the terminal condition for the present value model is not necessarily satisfied if we assume that only  $\frac{\Delta D}{D}$  and  $\frac{\Delta P}{P}$  are stationary. If the terminal condition is not satisfied, we have the undesirable result that there is no solution for the stock price. Shiller imposes the terminal condition by requiring the price-dividend ratio to be stationary.<sup>5</sup> The relationship in equation (1) is now modified as follows:

$$\frac{P_t^*}{D_t} = a^* + b^* \frac{P_t}{D_t} + \frac{\eta_t}{D_t},$$

and again  $a^*=0$  and  $b^*=1$  for the present value model.  $P_t^*$  is again a function of future dividends and we apply the same filter as before to both sides of the equation after rewriting it as follows:

$$P_t^* \equiv \frac{\beta L^{-1}}{1-\beta L^{-1}} D_t = a^* D_t + b^* P_t + \eta_t$$

$$D_t = a^* \left( \frac{1}{\beta} D_{t-1} - D_t \right) + b^* \left( \frac{1}{\beta} P_{t-1} - P_t \right) + \left( \frac{1}{\beta} \eta_{t-1} - \eta_t \right)$$

$$D_t = a D_{t-1} - b P_t + \frac{b}{\beta} P_{t-1} + \frac{-\eta_t + \frac{1}{\beta} \eta_{t-1}}{1+a^*},$$

where  $a = \frac{a^*}{\beta(1+a^*)}$  and  $b = \frac{b^*}{1+a^*}$ . Then equation can then be rewritten as a function of stationary time series plus an error term.

$$(4) \quad \frac{D_t}{D_{t-1}} = a - b\left(\frac{P_t}{D_{t-1}}\right) + \frac{b}{\beta}\left(\frac{P_{t-1}}{D_{t-1}}\right) + u_t,$$

where  $u_t = \frac{-\eta_t + \frac{1}{\beta}\eta_{t-1}}{(1+a^*)D_{t-1}}$ . Under the null hypothesis for the present value model,  $a=0$  and  $b=1$ . The error  $u_t$  again has the property that it is uncorrelated with variables dated  $t-1$  and earlier, but it is not possible to show that it is serially uncorrelated. Note that  $E(u_t(\frac{P_t}{D_{t-1}})) \neq 0$ , and again we must use an IV estimator. Because the error term may be serially correlated, we must use Hansen's (1982) generalized method of moments (GMM) estimator to estimate the parameters of equation (4). It is convenient to think of the GMM estimator in this case as an IV estimator generalized to allow for a serially correlated error term. The exact estimator used is presented in the next section.

The IV estimators for both cases (equations (3) and (4)) can be used to construct  $\chi^2$  tests of the restrictions for the present value model (a joint test that  $a=0$  and  $b=1$ ). This test has several advantages over the variance bounds test. The variance bounds test is based on an inequality restriction which may include a substantial amount of slack, whereas the test based on the IV estimators constitutes a direct test of the model. The IV estimation test has three additional properties: (1) an initial estimate of the discount factor is not required, (2) we do not need to assume that the fourth cumulants of the innovations are zero nor do we need to assume normality, and (3) we do not need to formulate a finite parameter time series model for dividends and stock prices.

### III. The IV and GMM Estimators

To develop the IV estimator for equation (3), first rewrite the equation in vector notation as follows:

$$\underline{u} = \underline{d} - a\underline{1} + b\underline{p} - \frac{b}{\gamma} \underline{p}_{-1},$$

where the vectors are of length T, the sample size, and  $\underline{1}$  is a vector of ones.  $\underline{p}_{-1}$  is the detrended price lagged one period. For the instrumental variables, we use a constant,  $d_{t-1}$ ,  $p_{t-1}$ , and  $x_{t-1}$  (detrended earnings lagged one period), so that the matrix of instrumental variables is

$$Z \equiv [ \underline{1} \quad \underline{d}_{-1} \quad \underline{p}_{-1} \quad \underline{x}_{-1} ].$$

Let  $\underline{\theta}' = (a, b, \gamma)$  and we estimate  $\underline{\theta}$  by<sup>6</sup>

$$\min_{\underline{\theta}} \ell = \underline{u}' Z (Z' Z)^{-1} Z' \underline{u}.$$

The covariance matrix for  $\underline{\theta}$  is computed as follows:

$$\text{Var}(\hat{\underline{\theta}}) = \hat{\sigma}_u^2 [ (\frac{\partial \underline{u}}{\partial \underline{\theta}})' Z (Z' Z)^{-1} Z' (\frac{\partial \underline{u}}{\partial \underline{\theta}}) ]^{-1},$$

where  $\frac{\partial \underline{u}}{\partial \underline{\theta}} \equiv [ \frac{\partial \underline{u}}{\partial a}, \frac{\partial \underline{u}}{\partial b}, \frac{\partial \underline{u}}{\partial \gamma} ]$  and

$$\hat{\sigma}_u^2 = \frac{1}{T} \sum_{t=1}^T (d_t - \hat{a} + \hat{b} p_t - \frac{\hat{b}}{\hat{\gamma}} p_{t-1})^2.$$

The restriction that  $a=0$  and  $b=1$  is tested by computing the following statistic:

$$\chi^2_{(2)} = (\hat{\underline{a}}_{b=1})' V^{-1} (\hat{\underline{a}}_{b=1}),$$

where  $V$  is the variance-covariance matrix for the estimates  $\hat{a}$  and  $\hat{b}$ . The test statistic has a large sample distribution that is Chi-squared with two degrees of freedom.

To estimate the parameters of equation (4), we must use Hansen's GMM estimator. To develop this estimator, first rewrite equation (4) as a function for  $u_t$ :

$$u_t = \frac{D_t}{D_{t-1}} - a + b \frac{P_t}{D_{t-1}} - \frac{b}{\beta} \frac{P_{t-1}}{D_{t-1}},$$

and let  $\underline{u}$  be a  $T \times 1$  vector containing  $u_t$ ,  $t=1, \dots, T$ . The instrumental variables for this estimator are included in a vector  $\underline{z}_t$ :

$$\underline{z}_t' = \left(1, \frac{D_{t-1}}{D_{t-2}}, \frac{P_{t-1}}{D_{t-1}}, \frac{X_{t-1}}{X_{t-2}}, \frac{P_{t-1}}{X_{t-1}}\right),$$

where  $X_t$  is earnings. Now form a matrix  $Z$  for the observations on the instruments so that  $\underline{z}_t'$  is the  $t$ 'th row of  $Z$ , a  $T \times 5$  matrix. Let  $\underline{\theta}' = (a, b, \beta)$  and form the GMM estimator as follows:

$$\min_{\underline{\theta}} \ell = \frac{1}{T} \underline{u}' Z W_T^{-1} Z' \underline{u},$$

where  $W_T$  is a weighting matrix which can be a function of sample data. Hansen shows that the optimal GMM estimator is the one in which  $W_T$  is equal to  $2\pi$  times the spectral density matrix for  $\underline{z}_t u_t$  evaluated at the zero frequency. In this paper, the matrix will be designated as  $S$ . In most cases, we must estimate this matrix. The following two-step estimator which replaces  $S$  with a consistent estimate  $\hat{S}$  is asymptotically equivalent to the optimal estimator: first, estimate  $\underline{\theta}$  using a weighting matrix which produces initial consistent estimates; then use the consistent estimates  $\hat{\underline{\theta}}$  to form  $u_t$  and estimate the spectrum of  $\underline{z}_t u_t$  to

produce a consistent estimate  $\hat{S}_T$ ; finally, estimate  $\underline{\theta}$  using  $\hat{S}_T$  as the weighting matrix. For the initial weighting matrix, I use  $W_T = \frac{1}{T} Z'Z$ , which yields the nonlinear two-stage least squares estimates which are consistent for  $\underline{\theta}$ . The smoothed periodogram and cross-periodogram estimators with flat windows are used to estimate  $S$ .<sup>7</sup> The second step of the estimator is

$$\min_{\underline{\theta}} \ell = \frac{1}{T} \underline{u}'Z \hat{S}_T^{-1} Z' \underline{u}.$$

The covariance matrix for  $\hat{\underline{\theta}}$  is

$$\text{Var}(\hat{\underline{\theta}}) = T[(\frac{\partial \underline{u}}{\partial \underline{\theta}})'Z \hat{S}_T^{-1} Z'(\frac{\partial \underline{u}}{\partial \underline{\theta}})]^{-1}.$$

The restriction that  $a=0$  and  $b=1$  is tested by computing a  $\chi^2$  statistic analogous to the previous one.

#### IV. Empirical Results

The present value model of stock prices is tested by using quarterly data for the Standard and Poor's Index of 500 Common Stocks for the period 1947 to the second quarter of 1983. In its publication Trade and Security Statistics, Standard and Poor's compiles quarterly data on its price index of 500 common stocks as well as indices on earnings and dividends for the companies included in the index.<sup>8</sup> The implicit price deflator for personal consumption expenditures is used to deflate the three series. The estimates for equations (3) and (4) are presented in Tables I and II, respectively. The equations have been estimated for two sample periods: the first sample period contains the longer period of 1947:I to 1983:II and the second sample period contains only the more recent period of 1960:I to 1983:II.

We must detrend the three series prior to estimating equation (3), and I have used a method similar to that used by Shiller. The following regressions are used to estimate the common growth trend for real dividends and real stock prices:

$$\ln D_t = \hat{a}_1 + \ln(\hat{1+g}_1) \cdot t,$$

$$\ln P_t = \hat{a}_2 + \ln(\hat{1+g}_2) \cdot t.$$

The two estimates,  $\ln(\hat{1+g}_1)$  and  $\ln(\hat{1+g}_2)$ , are averaged to form the OLS estimate of the common growth rate  $(1+g)$ , and this estimate is used to detrend both series. A separate trend regression is estimated for real earnings. For the longer sample period, the trend coefficient for  $\ln D_t$  is .004398 and for  $\ln P_t$  it is .005628. The resulting estimate of  $(1+g)$  is 1.005026. The results for the longer sample period are contained in the first column of Table I. Using the z statistics for a and b, one can reject the null hypotheses that  $a=0$  and  $b=1$  at standard significance levels: the z statistic for a is 12.18 and for the test that  $b=1$  it is -91.69. The  $\chi^2_{(2)}$  test statistic for the joint hypothesis has an extreme value of 12405, so that the marginal significance is effectively zero. For the more recent sample period, the estimates change, but the hypothesis tests lead to the same conclusion. The trend coefficient for real dividends is lower, and it is negative for stock prices. This result alone indicates that the data do not fit the model, or that stock prices deviated substantially from their long-term trend. The results for the more recent sample period are contained in the second column of Table I. The z statistic for a is 11.92 and the z statistic for the test that  $b=1$

is -159.95. The  $\chi^2_{(2)}$  test statistic has a value of 44539 so that the marginal significance is still effectively zero. The Durbin-Watson statistic is also quite low suggesting that the model is inadequate. These results are consistent with the previous results of the variance bounds tests, and it is clear that the test based on the IV estimator is much more powerful.

The results for equation (4) are equally damaging. For this equation, it is not necessary to detrend the data; the effects of long-term trends are removed when we form the ratios  $(\frac{D_t}{D_{t-1}}, \frac{P_t}{D_{t-1}}, \frac{P_{t-1}}{D_{t-1}})$ . The estimates for the longer period are presented in the first column and those for the more recent period are in the second column. The  $\chi^2_{(2)}$  statistic for the longer period is 5888 and for the more recent period it is 246356. The marginal significance level in both cases is effectively zero. The D.W. statistics support my concern that the error term in equation (4) is serially correlated. The tests for equation (4) indicate that the present value model is also rejected by the data if we model percentage changes in dividends and stock prices as stationary processes. These results do not substantiate this criticism of the previous work based on variance bounds tests.

## V. Conclusion

The results for the S&P 500 data suggest that the present value model of stock prices with rational expectations does not adequately describe the behavior of asset prices in the stock market. The alternative test described here does not require some of the additional assumptions required to derive tests based on direct estimates of

variances, and the results are much more conclusive than those previously obtained for tests of variance bounds. In addition, this test is modified to handle the case in which the percentage changes in real dividends and real stock prices are stationary. The null hypothesis that has been rejected is a joint hypothesis which includes the present value model of stock prices and rational expectations. Rejection implies that at least one part of the joint hypothesis has been rejected by the data. These tests cannot distinguish which part of the joint hypothesis has been rejected, and it is possible that both parts have been rejected. One alternative hypothesis is that expected real rates of return and real interest rates are not constant. Indeed, the results suggest that much of the variation in stock prices is due to something other than variation in dividends. This view of the stock market is consistent with observations that stock prices change when interest rates change even though there may be no new information to change expectations about future dividends and earnings.



## FOOTNOTES

1. For a discussion of this model and other expectations models, see LeRoy's survey (1982).
2. In most cases we must estimate the value of the discount factor and I have shown elsewhere (1982, pp. 114, 190-93) that the frequency domain variance estimator with a consistent estimate replacing the value of the discount factor is not asymptotically equivalent to the variance estimator which uses the true value of the discount factor.
3. For the asymptotic distribution of sample variances for time series, see Hannan (1970, pp. 209-12). In a footnote (footnote 15, p. 567), LeRoy and Porter recognize this subtle assumption in the derivation of their tests.
4. This subtle point is frequently ignored in applied econometrics. To apply Hansen's GMM estimator for example, we need to have stationary time series.
5. In a comment on Shiller's paper, Copeland (1983) has argued that the long term growth rate for dividends may be changing. His argument is confined to the effect of a one-period change, but we must consider how the growth rate changes over time. When we assume that the percentage changes in dividend and stock prices are stationary, we normally think of a constant growth rate for the series. If the random growth rate for dividends is itself a stationary process, then the series  $(\frac{\Delta D}{D}, \frac{\Delta P}{P})$  will be stationary. If we assume that the random growth rate is not stationary, then we again run the risk of not satisfying the terminal condition for the present value model.
6. This estimator is the nonlinear two-stage least squares estimator described in Amemiya (1974), which is also a special case of Hansen's GMM estimator. This nonlinear minimization problem can be easily solved by replacing  $b/\gamma$  with a new parameter, say  $h$ , and then solving the linear first order conditions for  $\hat{a}$ ,  $\hat{b}$ , and  $h$ .  $\hat{\gamma}$  is then computed as  $\hat{\gamma} = \hat{b}/\hat{h}$ .
7. For consistent estimators of the spectrum, see Hannan (1970, pp. 273-88), Nerlove, Grether, and Carvalho (1979, pp. 57-68), and Koopmans (1974, Ch. 8). Some sources call the estimator used here the Daniell estimator. I also used the Parzen procedure described in Nerlove, Grether, and Carvalho (p. 67). Specifically, fourth-order autoregressions were estimated for each series and the estimated filters were used to prewhiten the data. The spectral estimates were then recolored by using the transfer functions associated with the filters.
8. The data for dividends are reported as a twelve-month moving total. LeRoy and Porter obtained the data necessary to recompute the quarterly dividends and their numbers are available in a technical appendix to their original paper.

TABLE I

## RESULTS FOR ESTIMATION OF EQUATION (3)

$$d_t = a - b p_t + \frac{b}{\gamma} p_{t-1} + u_t$$

	<u>Sample Period</u> <u>1947:I to 1983:II</u>	<u>Sample Period</u> <u>1960:I to 1983:II</u>
$\hat{a}$	.4059	.9775
Standard error	(.03332)	(.08202)
$\hat{b}$	.01741	.007920
Standard error	(.01072)	(.006203)
$\hat{\gamma}$	.8998	1.092
Standard error	(.07415)	(.09944)
$\chi^2_{(2)}$	12405.	44539.
$\hat{\sigma}_u^2$	.01276	.01488
T	145	93
D.W.	2.06	.81
<u>Trend Coefficients</u>		
Dividends	.004398	.001158
Stock Price	.005628	-.005518
Earnings	.005572	.005267
$(1+\hat{g})$	1.005026	.9978225

TABLE II

RESULTS FOR ESTIMATION OF EQUATION (4)

$$\frac{D_t}{D_{t-1}} = a - b \frac{P_t}{D_{t-1}} + \frac{b}{\beta} \frac{P_{t-1}}{D_{t-1}} + u_t$$

	Sample Period <u>1947:I to 1983:II</u>	Sample Period <u>1960:I to 1983:II</u>
$\hat{a}$	.8095	.9492
Standard error	(.08783)	(.03752)
$\hat{b}$	.02249	.004810
Standard error	(.01548)	(.002007)
$\hat{\beta}$	.9089	.9033
Standard error	(.07598)	(.06250)
$\chi^2_{(2)}$	5888.	246356.
T	144	92
D.W.	3.50	2.85

## APPENDIX

To examine the properties of the error term in equation (3), we need to analyze the time series representation for  $d_t$  and  $p_t$  and the restrictions implied under the alternative hypothesis to the present value model. Let  $d_t$  have the following form in a linear multiple time series representation that is similar to equation (3) in LeRoy and Porter:

$$d_t = c + \underline{\delta}'_0 \underline{\varepsilon}_t + \underline{\delta}'_1 \underline{\varepsilon}_{t-1} + \dots = c + \underline{\delta}'(L) \underline{\varepsilon}_t.$$

Under the alternative hypothesis, we let market expectations be formed as follows:

$$\hat{E}_t(d_{t+j} - c) = \frac{1}{b} E_t(d_{t+j} - c),$$

where  $\hat{E}_t$  is the market's conditional expectation. The market systematically overestimates the deviation of dividends from its trend if  $b < 1$  and underestimates the deviation if  $b > 1$ . If  $b$  equals one, we have the present value model. The model now requires  $p_t$  to have the following time series representation:

$$\begin{aligned} p_t &= \hat{E}_t \sum_{j=1}^{\infty} \gamma^j d_{t+j} = \frac{c\gamma(b-1)}{(1-\gamma)b} + \frac{1}{b} E_t \sum_{j=1}^{\infty} \gamma^j d_{t+j} \\ &= \frac{c\gamma}{1-\gamma} + \frac{1}{b} [\underline{a}'_0 \underline{\varepsilon}_t + \underline{a}'_1 \underline{\varepsilon}_{t-1} + \dots] \\ &= \frac{c\gamma}{1-\gamma} + \frac{1}{b} \underline{a}'(L) \underline{\varepsilon}_t, \end{aligned}$$

where  $\underline{a}'_j = \sum_{k=1}^{\infty} \gamma^k \underline{\delta}'_{k+j}$ . We now examine the error term  $u_t$ . The intercept  $a$  in equation (3) equals  $c(1-b)$ .

$$\begin{aligned} u_t &= d_t - a + b p_t - \frac{b}{\gamma} p_{t-1} \\ &= c + \underline{\delta}'(L) \underline{\varepsilon}_t - c(1-b) + \frac{c\gamma b}{1-\gamma} + \underline{a}'(L) \underline{\varepsilon}_t - \frac{cb}{1-\gamma} - \frac{1}{\gamma} \underline{a}'(L) \underline{\varepsilon}_{t-1}. \end{aligned}$$

It can be easily shown that all the constants on the right hand side of the equation cancel. Next, we match the coefficients on  $\underline{\varepsilon}_{t-1}$ ,  $\underline{\varepsilon}_{t-2}$ , ... and show

$$\begin{aligned} \underline{\delta}'_j \underline{\varepsilon}_{t-j} + \underline{a}'_j \underline{\varepsilon}_{t-j} - \frac{1}{\gamma} \underline{a}'_{j-1} \underline{\varepsilon}_{t-j} &= \left( \underline{\delta}'_j + \sum_{k=1}^{\infty} \gamma^k \underline{\delta}'_{k+j} - \frac{1}{\gamma} \sum_{k=1}^{\infty} \gamma^k \underline{\delta}'_{k+j-1} \right) \underline{\varepsilon}_{t-j} \\ &= 0 \quad \text{for } j = 1, 2, 3, \dots \end{aligned}$$

Hence, the coefficients on  $\underline{\varepsilon}_{t-1}$ ,  $\underline{\varepsilon}_{t-2}$ , .... are all zero and we have for the error term:

$$u_t = \left( \underline{a}'_0 + \underline{\delta}'_0 \right) \underline{\varepsilon}_t = \left( \sum_{k=0}^{\infty} \gamma^k \underline{\delta}'_k \right) \underline{\varepsilon}_t.$$

The error term is a linear combination of only the current innovations of the dividend series and is therefore serially uncorrelated. If we let  $b=1$ , then the analysis also applies for the present value model.

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